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Excited K-quantum nonlinear coherent states

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Abstract

We define the excited K-quantum nonlinear coherent state and prove that it is indeed a K-quantum nonlinear coherent state characterized by another nonlinear function whose general expression is found in an explicit form. We also demonstrate that antibunching is more prominent in the excited K-quantum nonlinear coherent state than in the corresponding K-quantum nonlinear coherent state.

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A large number of nonclassical states (see, e.g., [1–5]) have been recognized so far. They are, however, not all independent. For example, the so-called photon-added coherent state [6,7] has recently been shown [8] to belong to the nonlinear coherent state (NCS) [9, 10] and the excited NCS has been proved [11] to remain a NCS. The q-deformed oscillator [12, 13] was known just as a particular case of the more general f oscillator [10, 14] whose eigenstate is nothing other than the NCS. The NCS also embraces the negative binomial state [11, 15], etc. Re-classification of nonclassical states proves of great interest in understanding and distinguishing the physics underlying such states.

Recently the K-quantum nonlinear coherent state (KNCS) has been introduced, whose generation schemes as well as various nonclassical properties such as multi-peaked number distribution, self-splitting, antibunching and squeezing, have been studied in detail in [16–19]. The KNCS, $|\xi, K, j, f\rangle$, is defined as the right eigenstate of the non-Hermitian operator $a^K f(\hat{n})$:

$$a^K f(\hat{n})|\xi, K, j, f\rangle = \xi^K |\xi, K, j, f\rangle, \tag{1}$$

with a the boson annihilation operator, f a nonlinear operator-valued function of $\hat{n} = a^+a$, ξ the complex eigenvalue, K a positive integer and $j = 0, 1, \ldots, K-1$. This kind of state includes the Peremolov K-photon coherent state [20, 21].

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In this brief paper we shall present a rigorous proof that the excited KNCS, i.e. the state generated by the excitations on a KNCS, is also a KNCS. That is, exciting a KNCS does not transform it to another class of state. This is in contrast to the situation of excitations on a coherent state.

In terms of the KNCS characterized by the function $f(\hat{n})$ we define the excited KNCS by acting by $(a^+)^M$ on it. The state is thus

$$(a^+)^M|\xi, K, j, f\rangle \tag{2}$$

with M a positive integer. We shall prove that such an excited KNCS also belongs to the family of the KNCS. That is, it is another KNCS which is characterized by the same K but with some other nonlinear function $f_{K,M}$ different from f and dependent on both K and M. To determine the $f_{K,M}$ we distinguish between two situations $M \geqslant K$ and $M \leqslant K$.

For $M \geqslant K$ we can write $M = K + m$ with $m = M - K \geqslant 0$. Acting with $(a^+)^M$ on both sides of (1) gives

$$(a^+)^M a^K f(\hat{n})|\xi, K, j, f\rangle = (a^+)^M \xi^K|\xi, K, j, f\rangle. \tag{3}$$

The RHS of (3) is just the excited KNCS (2) multiplied by ξ^K:

$$\mathrm{RHS} = \xi^K (a^+)^M|\xi, K, j, f\rangle. \tag{4}$$

On the LHS of (3) we represent $(a^+)^M$ as $(a^+)^m(a^+)^K$ and use the identity [22]

$$(a^+)^K a^K = \prod_{l=0}^{K-1}(\hat{n} - l) \tag{5}$$

to cast it into

$$\mathrm{LHS} = (a^+)^m \prod_{l=0}^{K-1}(\hat{n} - l) f(\hat{n})|\xi, K, j, f\rangle. \tag{6}$$

Because of the identity

$$(a^+)^m g(\hat{n}) = g(\hat{n} - m)(a^+)^m \qquad \forall g(\hat{n}), m \tag{7}$$

the expression (6) becomes

$$\mathrm{LHS} = \prod_{l=0}^{K-1}(\hat{n} - m - l) f(\hat{n} - m)(a^+)^m|\xi, K, j, f\rangle. \tag{8}$$

To proceed further we note the identity [22]

$$a^K (a^+)^K = \prod_{k=1}^{K}(\hat{n} + k) \tag{9}$$

which allows us to write the unity operator $\hat{1}$ in the form

$$\hat{1} = \frac{1}{\prod_{k=1}^{K}(\hat{n} + k)} a^K (a^+)^K. \tag{10}$$

Inserting this operator $\hat{1}$ in between $f(\hat{n} - m)$ and $(a^+)^m$ in (8), upon use of

$$g(\hat{n}) a^K = a^K g(\hat{n} - K) \qquad \forall g(\hat{n}), K, \tag{11}$$

then becomes

$$\mathrm{LHS} = a^K f(\hat{n} - M) \frac{\prod_{l=0}^{K-1}(\hat{n} - M - l)}{\prod_{k=1}^{K}(\hat{n} - K + k)} (a^+)^M|\xi, K, j, f\rangle. \tag{12}$$

From (4) and (12) we see by virtue of (1) that the excited KNCS (2) is in fact a KNCS:

$$(a^{+})^{M}|\xi, K, j, f\rangle = |\xi, K, j, f^{>}_{K,M}\rangle \tag{13}$$

with the new K- and M-dependent nonlinear function $f^{>}_{K,M}$ given by

$$f^{>}_{K,M}(\hat{n}) = f(\hat{n}-M)\frac{\prod_{l=0}^{K-1}(\hat{n}-M-l)}{\prod_{k=1}^{K}(\hat{n}-K+k)}. \tag{14}$$

For $M \leqslant K$ we write $K = M + m$ with $m = K - M \geqslant 0$. Acting with $(a^{+})^{M}$ on both sides of (1) we have the same RHS as in (4) but the LHS in this case is

$$\mathrm{LHS} = (a^{+})^{M} a^{K} f(\hat{n})|\xi, K, j, f\rangle = (a^{+})^{M} a^{M} a^{m} f(\hat{n})|\xi, K, j, f\rangle. \tag{15}$$

Using again the identities (5) and (11) yields

$$\mathrm{LHS} = \prod_{l=0}^{M-1}(\hat{n}-l) a^{m} f(\hat{n})|\xi, K, j, f\rangle = a^{m} f(\hat{n}) \prod_{l=0}^{M-1}(\hat{n}-m-l)|\xi, K, j, f\rangle. \tag{16}$$

Inserting in between a^{m} and $f(\hat{n})$ the unity operator $\hat{1}$, which is constructed this time as

$$\hat{1} = \frac{1}{\prod_{k=1}^{M}(\hat{n}+k)} a^{M}(a^{+})^{M}, \tag{17}$$

we get (16) in the form

$$\mathrm{LHS} = a^{m}\frac{1}{\prod_{k=1}^{M}(\hat{n}+k)} a^{M}(a^{+})^{M} f(\hat{n}) \prod_{l=0}^{M-1}(\hat{n}-m-l)|\xi, K, j, f\rangle. \tag{18}$$

Applying the identities (11) and (7) to (18) casts it into

$$\mathrm{LHS} = a^{K} f(\hat{n}-M)\frac{\prod_{l=0}^{M-1}(\hat{n}-K-l)}{\prod_{k=1}^{M}(\hat{n}-M+k)}(a^{+})^{M}|\xi, K, j, f\rangle. \tag{19}$$

From (4) and (19) we see by virtue of (1) that also in this situation the excited KNCS (2) is a KNCS:

$$(a^{+})^{M}|\xi, K, j, f\rangle = |\xi, K, j, f^{<}_{K,M}\rangle \tag{20}$$

with another K- and M-dependent nonlinear function $f^{<}_{K,M}$ given by

$$f^{<}_{K,M}(\hat{n}) = f(\hat{n}-M)\frac{\prod_{l=0}^{M-1}(\hat{n}-K-l)}{\prod_{k=1}^{M}(\hat{n}-M+k)}. \tag{21}$$

Combining the two expressions (14) and (21) derived separately in the two situations mentioned above leads us to the joined result valid for arbitrary positive integers M and K: the excited KNCS, $(a^{+})^{M}|\xi, K, j, f\rangle$, is nothing other than a KNCS $|\xi, K, j, f_{K,M}\rangle$ with the same K but with the nonlinear function $f_{K,M}$, other than f, which depends on both K and M as

$$f_{K,M}(\hat{n}) = f(\hat{n}-M)\frac{\prod_{l=0}^{L_{\min}-1}(\hat{n}-L_{\max}-l)}{\prod_{k=1}^{L_{\min}}(\hat{n}-L_{\min}+k)} \tag{22}$$

where $L_{\max} = \max(M, K)$ and $L_{\min} = \min(M, K)$. It seems at first glance from (22) that different formulae apply to different situations ($M \geqslant K$ or $M \leqslant K$). However, we find out that a beautiful expression can be obtained if we notice that

$$\prod_{l=0}^{L_{\min}-1}(\hat{n}-L_{\max}-l) = \frac{(\hat{n}-L_{\max})!}{(\hat{n}-L_{\max}-L_{\min})!} \tag{23}$$

and

$$\prod_{k=1}^{L_{\min}}(\hat{n} - L_{\min} + k) = \frac{\hat{n}!}{(\hat{n} - L_{\min})!}. \tag{24}$$

Substituting (23) and (24) into (22) we arrive at

$$f_{K,M}(\hat{n}) = f(\hat{n} - M)\frac{(\hat{n} - L_{\max})!(\hat{n} - L_{\min})!}{\hat{n}!(\hat{n} - L_{\max} - L_{\min})!} \tag{25}$$

or, more explicitly,

$$f_{K,M}(\hat{n}) = f(\hat{n} - M)\frac{(\hat{n} - K)!(\hat{n} - M)!}{\hat{n}!(\hat{n} - K - M)!}. \tag{26}$$

As it stands, this formula (26) applies to arbitrary M and K, no matter that $M \geqslant K$ or $M \leqslant K$.

Because our derivation holds true for any positive integers M and K, it applies as well to the case of $K = 1$ for which we obtain from (26)

$$f_{1,M}(\hat{n}) = f(\hat{n} - M)\left(1 - \frac{M}{\hat{n}}\right) \tag{27}$$

which agrees with the result reported in [11]. If $f \equiv 1$, (27) reduces further to

$$f_{1,M}(\hat{n}) = \left(1 - \frac{M}{\hat{n}}\right) \tag{28}$$

which corresponds to the case of photon-added coherent states [6, 7] as discovered in [8]. For $K = 2$

$$f_{2,M}(\hat{n}) = f(\hat{n} - M)\left(1 - \frac{M(M + 1 - 2\hat{n})}{\hat{n}(\hat{n} - 1)}\right), \tag{29}$$

and for $K = 3$

$$f_{3,M}(\hat{n}) = f(\hat{n} - M)\left(1 - \frac{M(M^2 + 3M + 2 - 3(M + 2)\hat{n} + 3\hat{n}^2)}{\hat{n}(\hat{n} - 1)(\hat{n} - 2)}\right), \tag{30}$$

and so on.

Since it is well known that the quantum statistical properties of the KNCS are very sensitive to the nonlinear function, the excited KNCS promises novel behaviour as compared to the original KNCS because the excitation modifies the nonlinear function from $f(\hat{n})$ to $f_{K,M}(\hat{n})$. The clearest example concerns the case of $K = 1$ and $f(\hat{n}) = 1$: for $M = 0$ it is the coherent state which is purely classical, whereas for $M \geqslant 1$ it is the photon-added coherent state which is intrinsically nonclassical. To emphasize the significance of the results, let us now examine the dependence on the excitation level of the antibunching character of the vibrational motion of a laser-driven trapped ion in an excited KNCS with $K > 1$. Without excitation, i.e. with $M = 0$, the concerned KNCS is characterized by the nonlinear function [18, 23]

$$f(\hat{n}) = \frac{(\hat{n} - K)!L^K_{\hat{n}-K}(\eta^2)}{\hat{n}L^0_{\hat{n}-K}(\eta^2)} \tag{31}$$

where η is the Lamb–Dicke parameter and $L^q_n(z)$ is the nth generalized Laguerre polynomial in z for a parameter q. Antibunching appears if $A_{Kj} < 1$ where

$$A_{Kj} = \frac{\langle(a^+a)^2\rangle - \langle a^+a\rangle^2}{\langle a^+a\rangle} \tag{32}$$

with $\langle\cdots\rangle \equiv \langle f_{K,M}, j, K, \xi|\cdots|\xi, K, j, f_{K,M}\rangle$ the underlying quantum average. In figure 1 we plot A_{30}, A_{31} and A_{32} as a function of $|\xi|$ for different levels of excitation. This figure shows

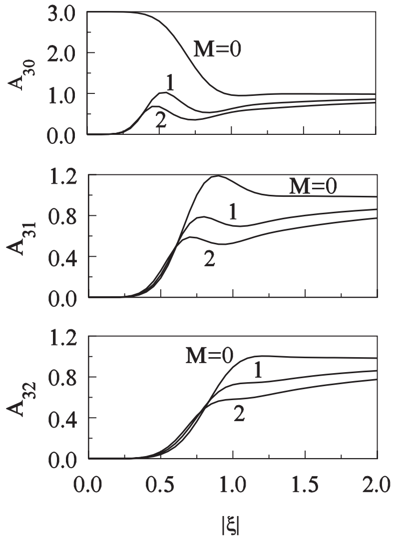

Figure 1. A_{30}, A_{31} and A_{32} in the excited KNCS as a function of $|\xi|$ for $\eta = 0.05$ and different M. The value of M is indicated near the curve.

that the higher the excitation level, i.e. the greater the value of M, the better the antibunching degree. More interestingly, we also observe that antibunching always exists in an excited KNCS ($M \geqslant 1$) but may be absent in the original KNCS ($M = 0$) in some range of $|\xi|$, as can be clearly seen from figure 1 for A_{30} and A_{31}. Exciting a KNCS thus provides a means to manipulate the quantum state. This would give relevant motivation for more thorough studies in the future.

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References

[1] Helstrom C W 1980 *IEEE Trans. Inf. Theory* **26** 378
[2] Loudon R 1980 *Rep. Prog. Phys.* **43** 913
[3] Hillery M 1985 *Phys. Rev.* A **31** 338
[4] Mandel L 1986 *Phys. Scr.* T **12** 34
[5] Dodonov V V 2002 *J. Opt. B: Quantum Semiclass. Opt.* **4** R1
[6] Agarwal G S and Tara K 1991 *Phys. Rev.* A **43** 492
[7] Dodonov V V, Korennoy Ya A, Man'ko V I and Moukhin Y A 1996 *Quantum Semiclass. Opt.* **8** 413
[8] Sivakumar S 1999 *J. Phys. A: Math. Gen.* **32** 3441
[9] de Matos Filho R L and Vogel W 1996 *Phys. Rev.* A **54** 4560
[10] Man'ko V I, Marmo G, Sudarshan E C G and Zaccaria F 1997 *Phys. Scr.* **55** 528
[11] Wang X G and Fu H C 1999 *Mod. Phys. Lett.* B **13** 617
[12] Biedenharn L C 1989 *J. Phys. A: Math. Gen.* **22** L873
[13] Macfarlane A J 1989 *J. Phys. A: Math. Gen.* **22** 4581
[14] Man'ko V I, Marmo G, Solimeno S and Zaccaria F 1993 *Int. J. Mod. Phys.* A **8** 3577
[15] Fu H C and Sasaki R 1997 *J. Phys. Soc. Japan* **66** 1989
[16] Liu X M 1999 *J. Phys. A: Math. Gen.* **32** 8685

[17] Man'ko V I, Marmo G, Porzio A, Solimeno A and Zaccaria F 2000 *Phys. Rev.* A **62** 053407
[18] Nguyen B A 2001 *Chin. J. Phys.* **39** 594
[19] Nguyen B A 2001 *Phys. Lett.* A **284** 72
[20] D'Ariano G, Rasetti M and Vadacchino M 1985 *Phys. Rev.* D **32** 1034
[21] Katriel J, Rasetti M and Solomon A I 1987 *Phys. Rev.* D **35** 1284
[22] Nguyen B A 1997 *Phys. Lett.* A **234** 45
[23] Nguyen B A and Truong M D 2002 *Int. J. Mod. Phys.* B **16** 519